Qual é o comprimento da costa do Brasil?

Bernardo Xavier

Colégio de Aplicação - UFRJ

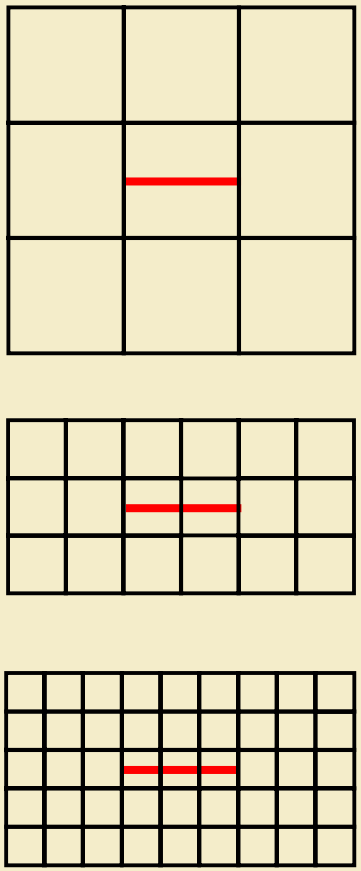
Orientador: C. E. Aguiar

Instituto de Física - UFRJ

Roteiro

- **O que é Dimensão**
- **Fractais**
- **Um programa para medir a dimensão fractal**
- **Aplicações**

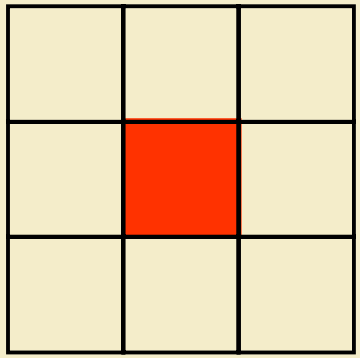
O que é Dimensão (de Caixa)

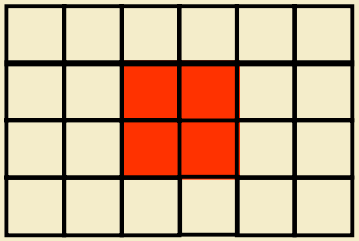


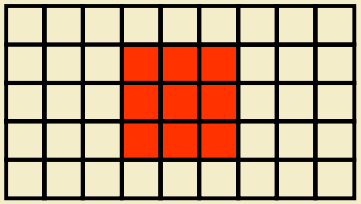
Número de caixas:	Comprimento das caixas:
N=1	**L=1**
N=2	**L=1/2**
N=3	**L=1/3**

$$N = L^{-1} \Rightarrow D = 1$$

O que é Dimensão (de Caixa)



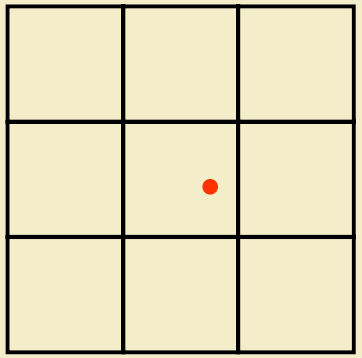


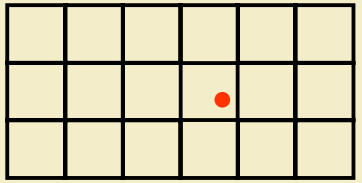


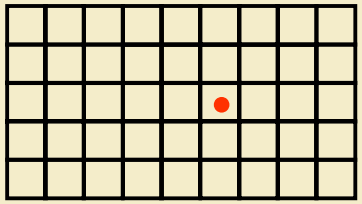
Número de caixas:	Comprimento das caixas:
N=1	**L=1**
N=4	**L=1/2**
N=9	**L=1/3**

$$N = L^{-2} \Rightarrow D = 2$$

O que é Dimensão (de Caixa)







Número de caixas:	Comprimento das caixas:
N=1	**L=1**
N=1	**L=1/2**
N=1	**L=1/3**

$$N = L^{0} \Rightarrow D = 0$$

O que é Dimensão (de Caixa)

Figura coberta por N caixas de comprimento L:

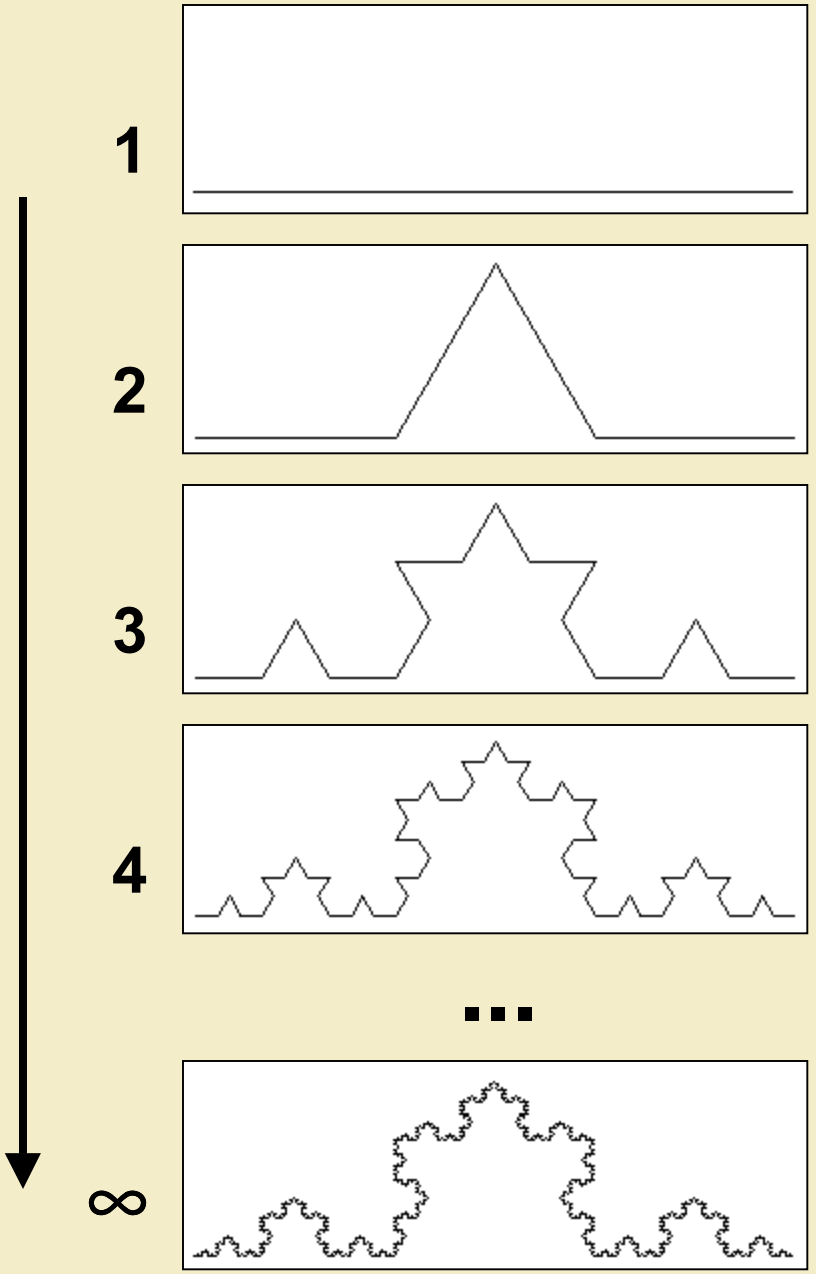
$$N = L^{-D}$$

D = dimensão de caixa

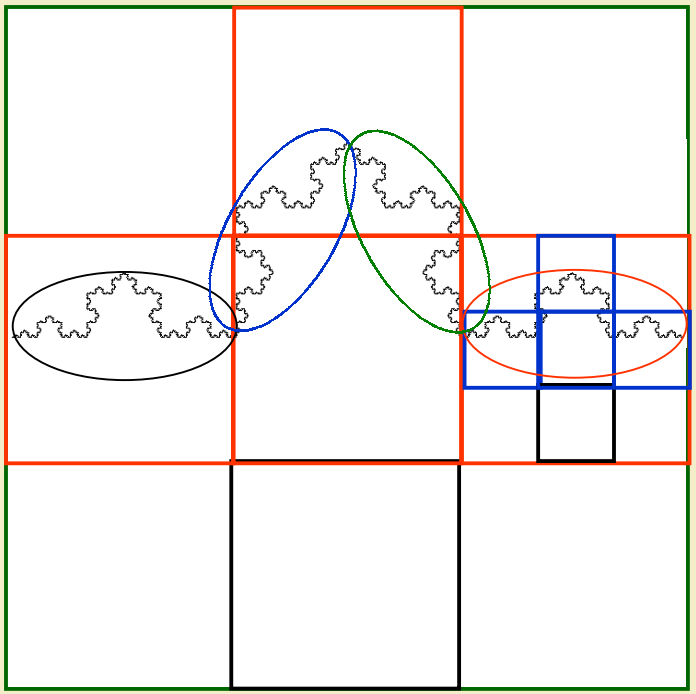
Fractais

A Curva de Koch

Construção:



Qual é a Dimensão da Curva de Koch?



Número de caixas:	Comprimento das caixas:
N=1	**L=1**
N=4	**L=1/3**
N=16	**L=1/9**

$$N = L^{-D} \quad \Rightarrow \quad D = (\log N)/(\log L^{-1})$$

$$D = (\log 4)/(\log 3) = 1{,}26...$$

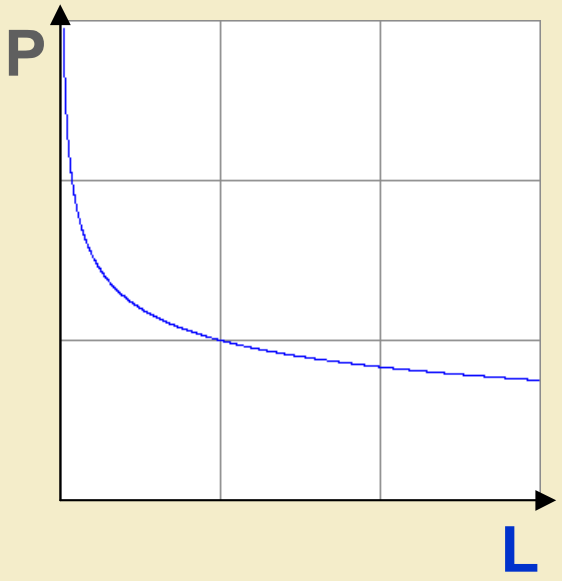
Qual é o Comprimento da Curva de Koch?

Comprimento: $P \sim N \times L \sim L^{1-D}$

$P \sim L^{1-D}$

$P \sim L^{1-1,26}$

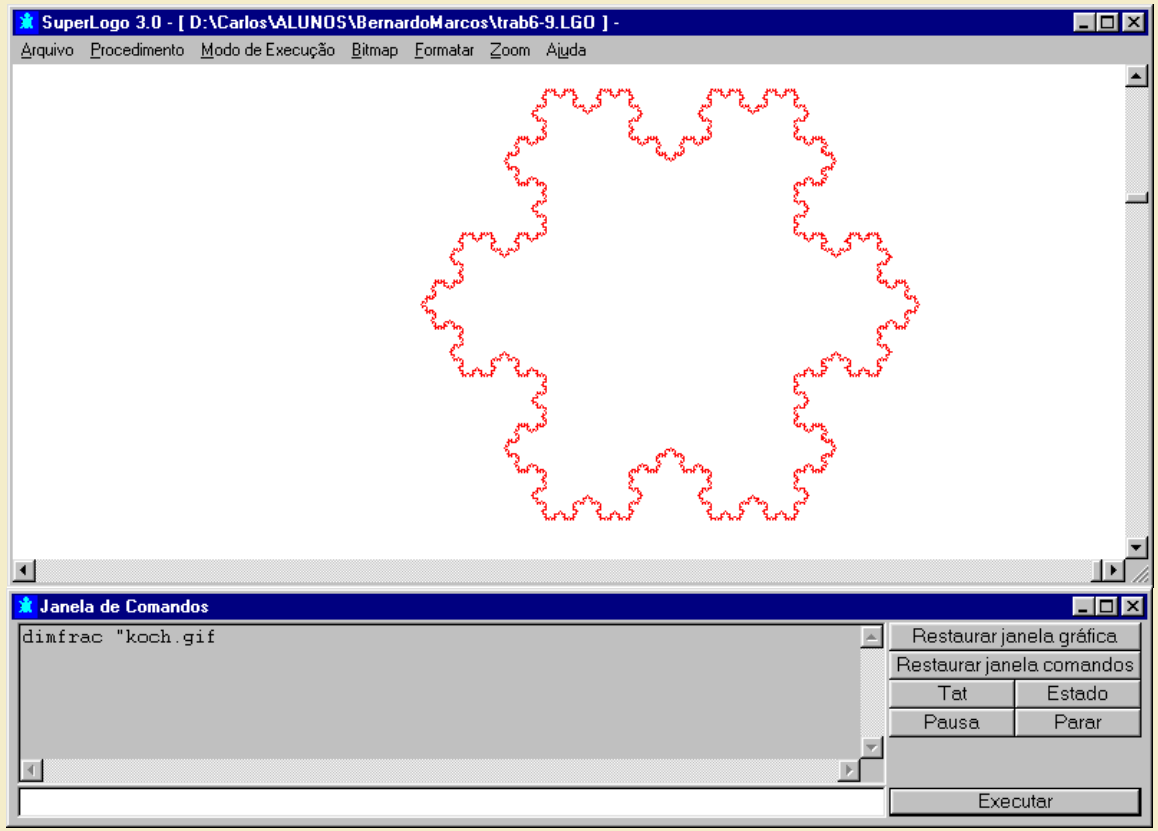
$P \sim L^{-0,26}$



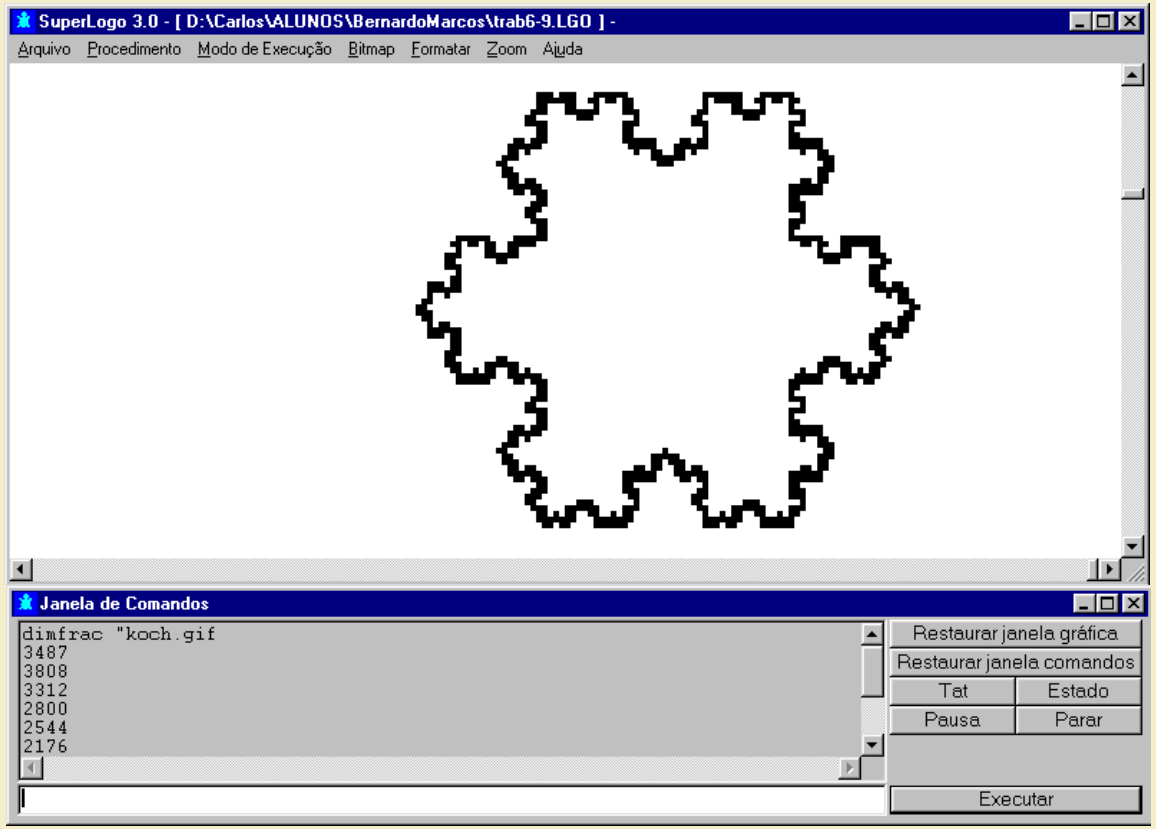
L	P
1	1,00
1/2	1,20
1/3	1,33
1/4	1,44
1/5	1,52

$L \rightarrow 0 \Rightarrow P \rightarrow \infty$

Um Programa Logo para Medir a Dimensão Fractal

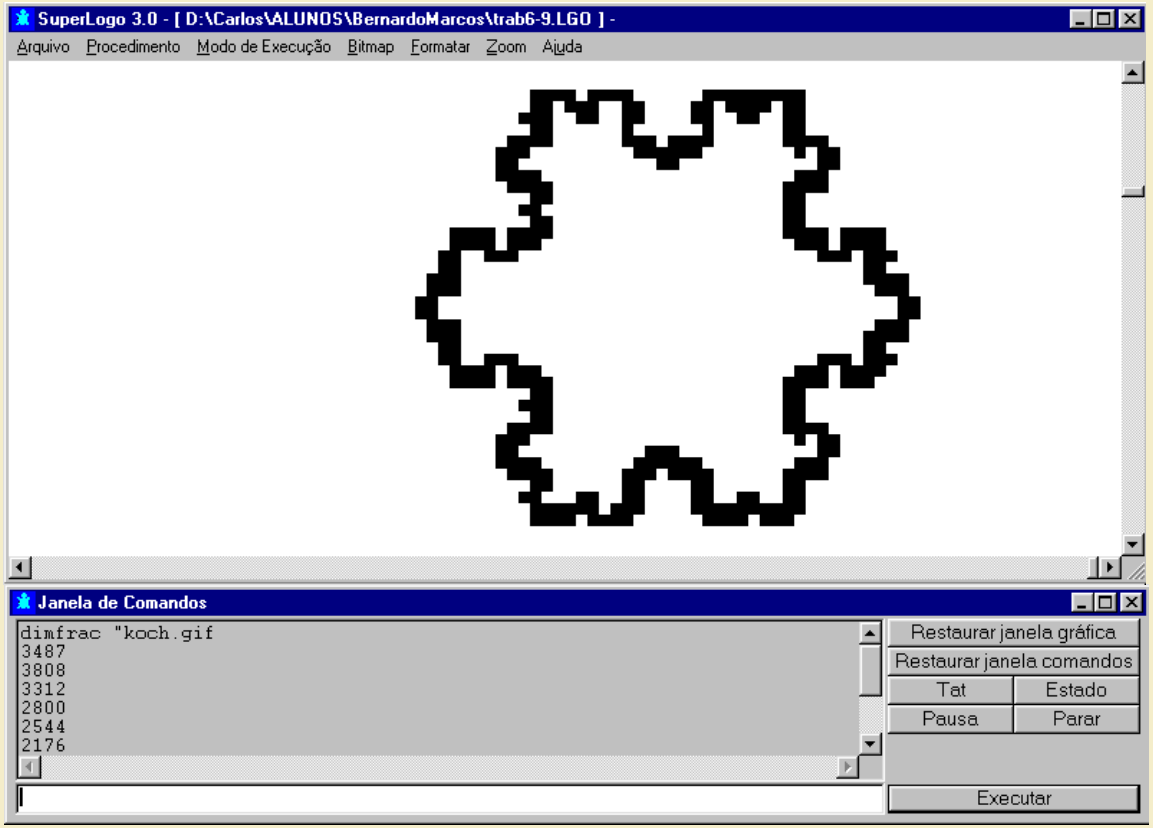


Um Programa Logo para Medir a Dimensão Fractal



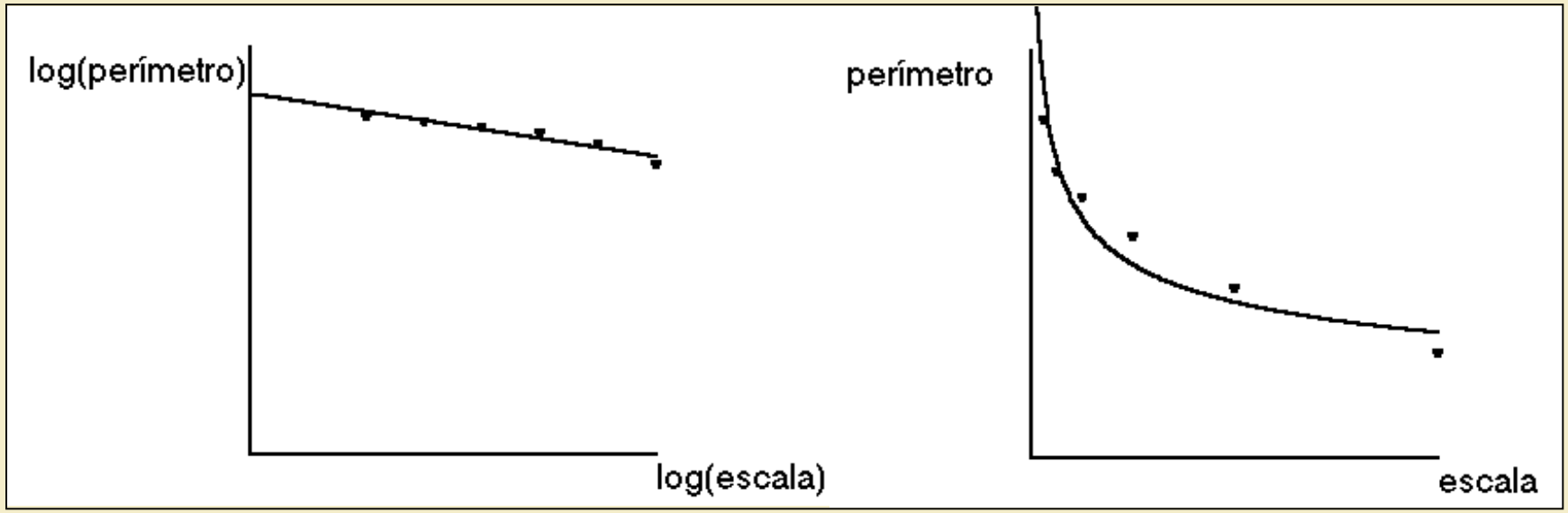
L = 4 pxl
N = 828
P =3312

Um Programa Logo para Medir a Dimensão Fractal



L = 8 pxl
N = 350
P =2800

Um Programa Logo para Medir a Dimensão Fractal



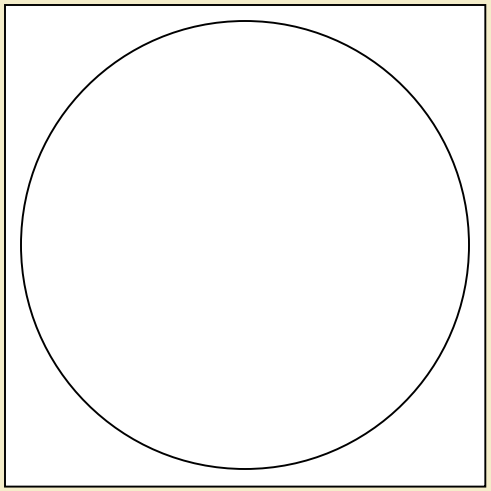
$\log P = (1-D) \log L + cte$

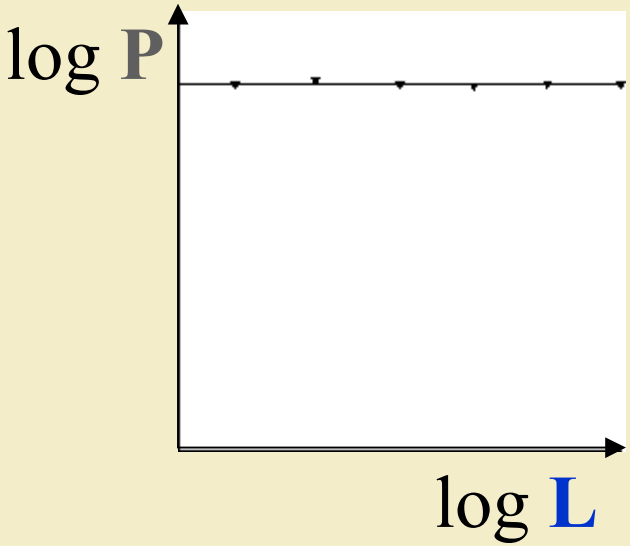
$P = cte\ L^{1-D}$

D = 1,29 ± 0,04

D(exato) = 1,2618...

Dimensão da Circunferência

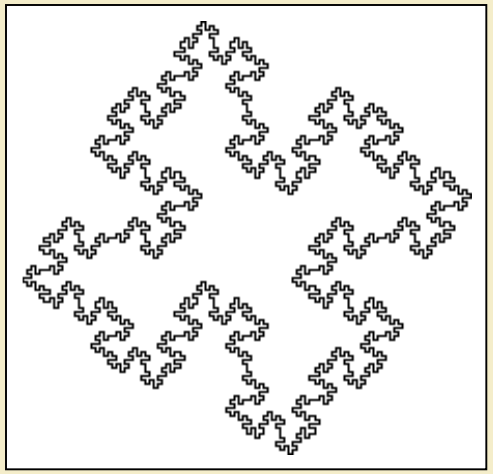


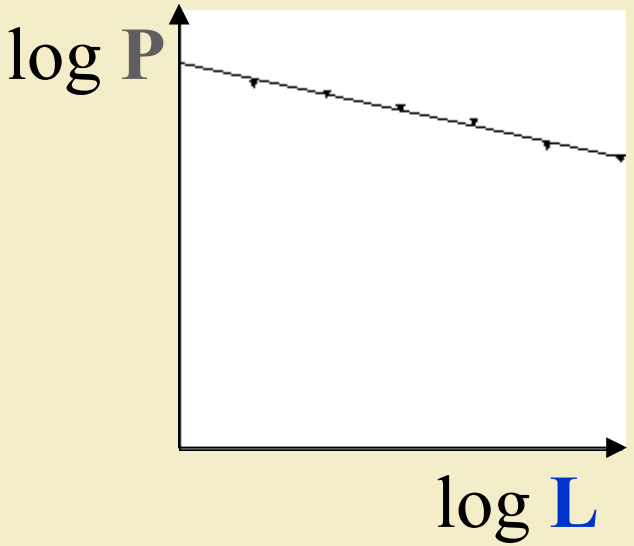


D = 0,99 ± 0,02

D(exato) = 1

Ilha de Koch Quadrangular

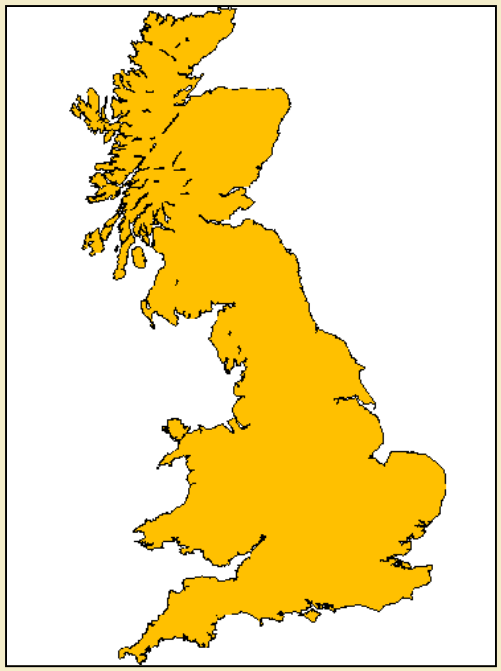


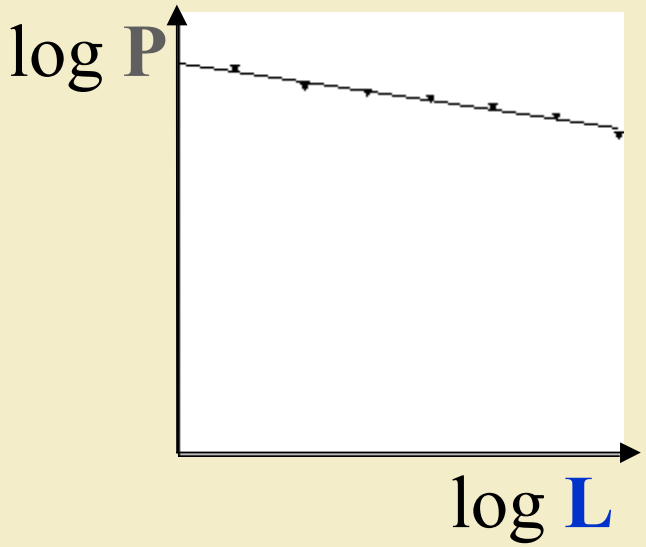


$D = 1{,}51 \pm 0{,}03$

$D(\text{exato}) = 1{,}5$

Costa da Grã-Bretanha

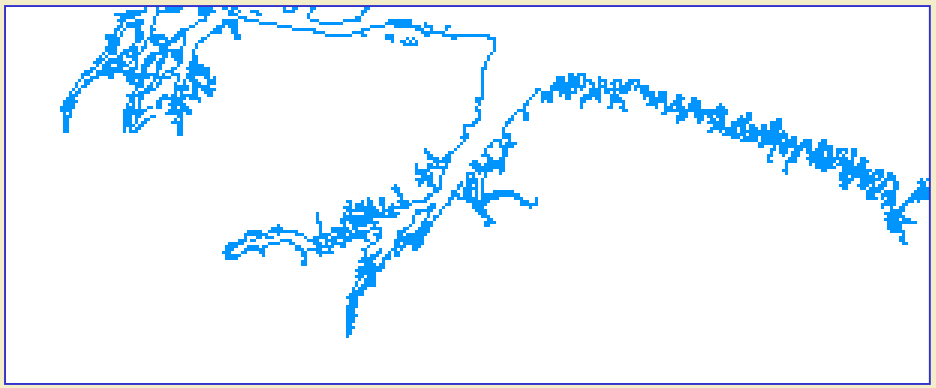


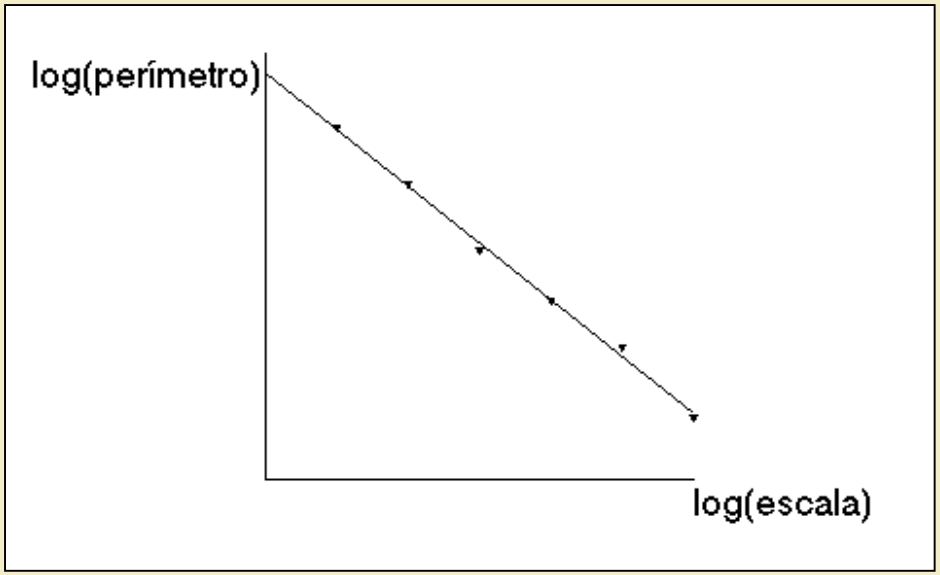


D = 1,30 ± 0,04

D = 1,31 (Peitgen et al.)

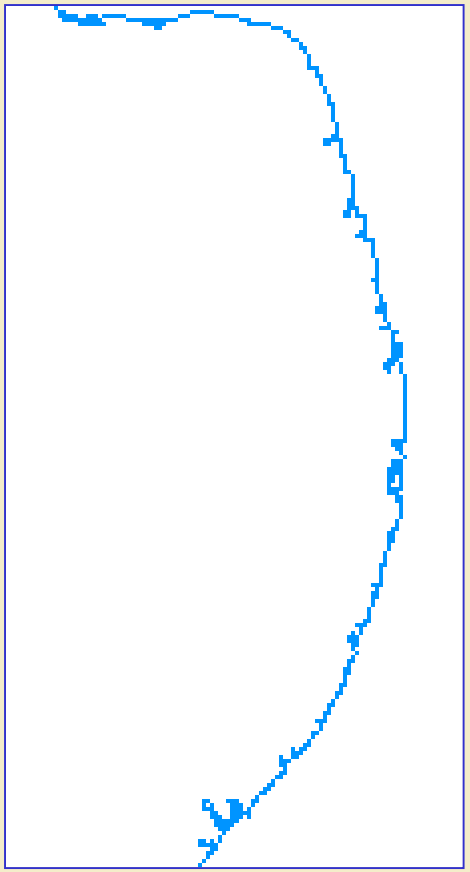
Costa do Pará-Maranhão

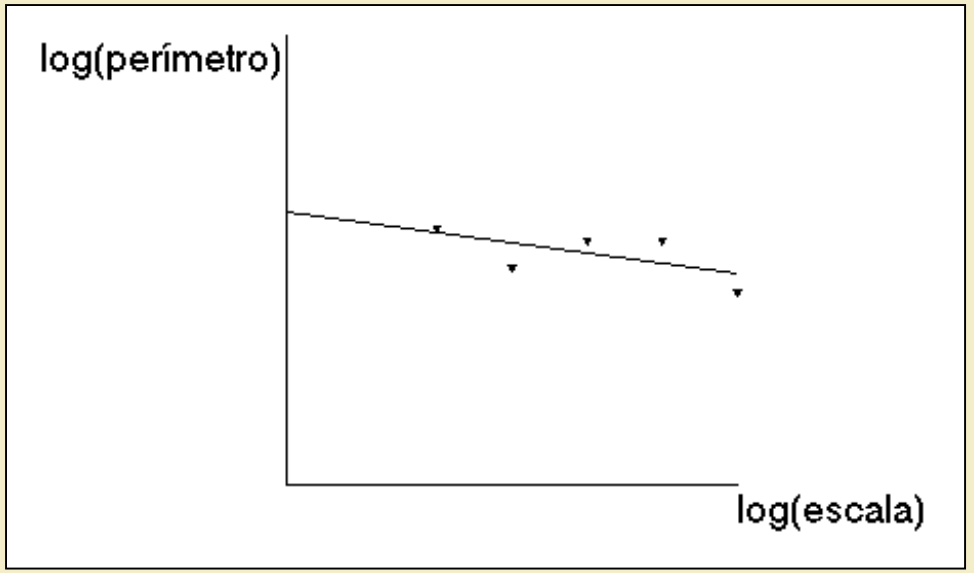




$$D = 1{,}41 \pm 0{,}01$$

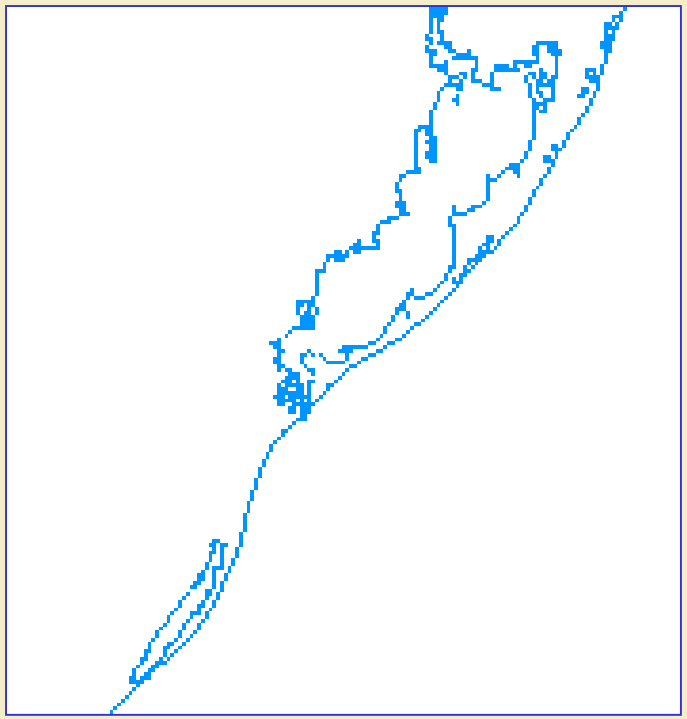
Costa do Nordeste

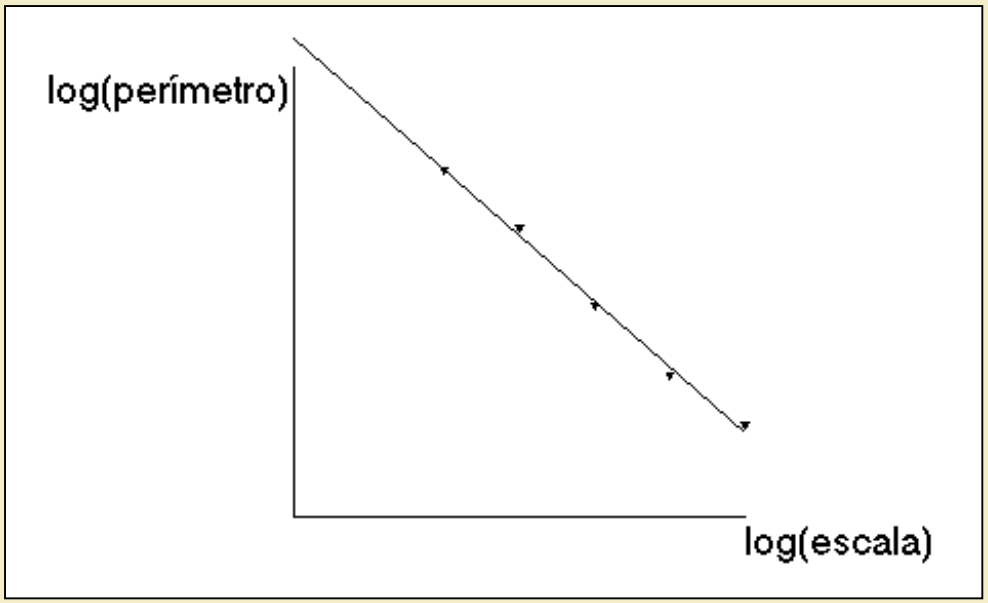




D = 1,02 ± 0,02

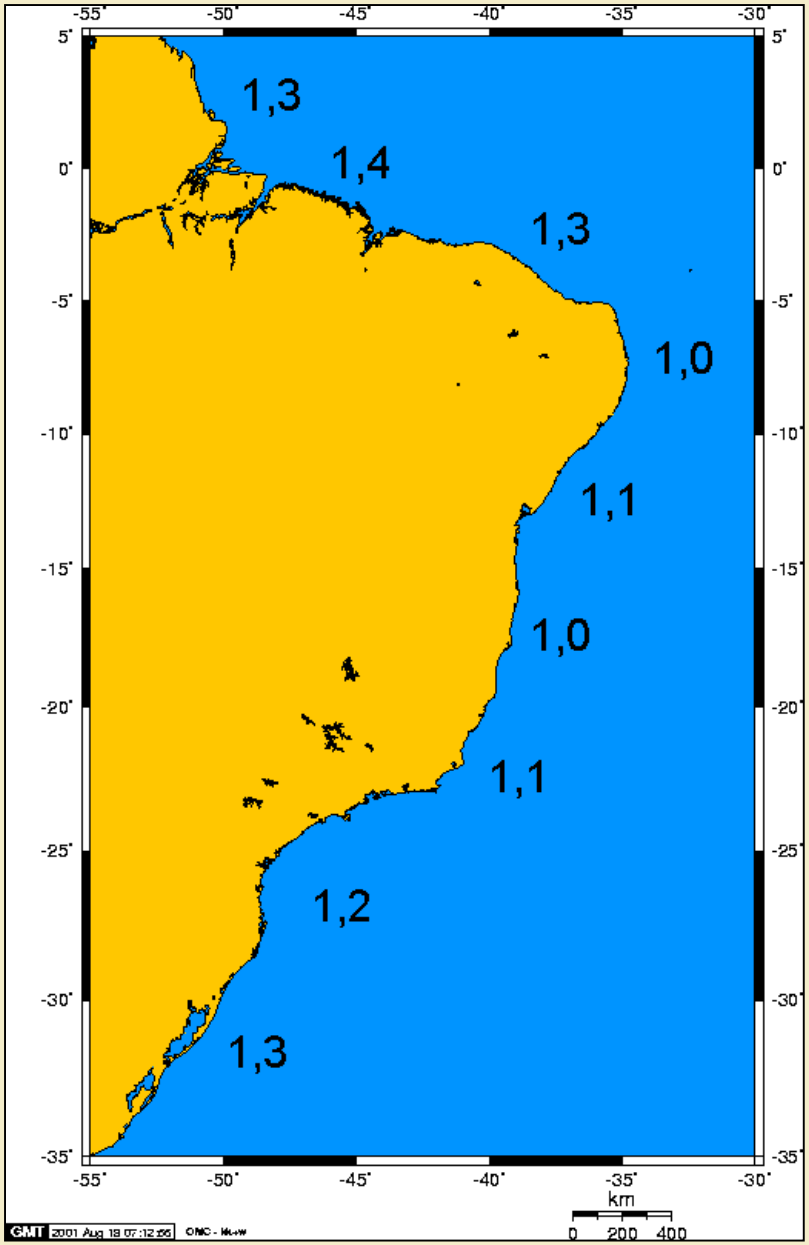
Costa do R. G. do Sul



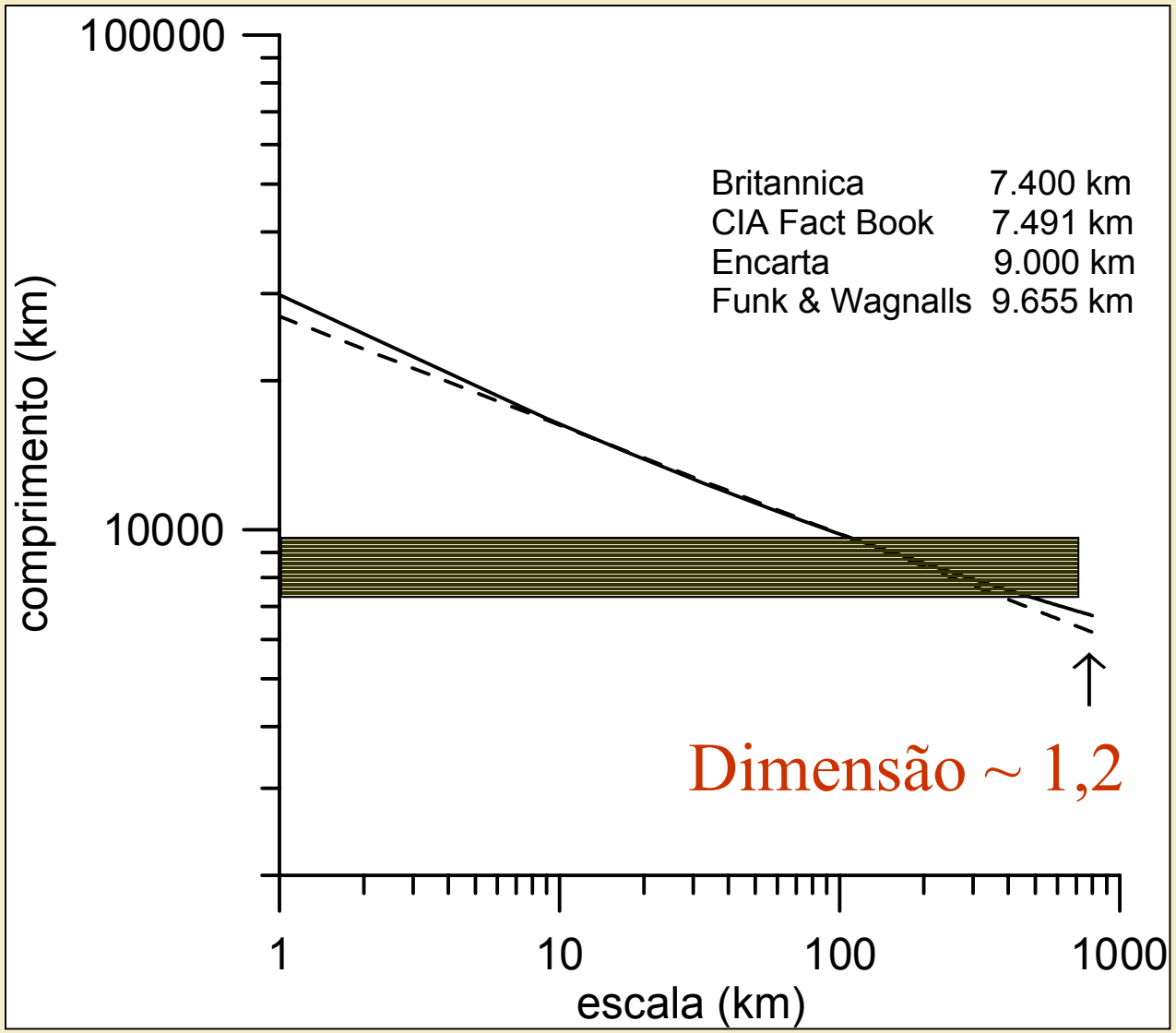


D = 1,30 ± 0,01

Costa do Brasil



Qual o Comprimento da Costa do Brasil?



Concluindo:

- A costa brasileira é um fractal, logo não tem um (único) comprimento. O comprimento depende da escala de medida.

- Comprimento = 14.000 km $\leftrightarrow$ escala de 20 km
  12.000 km $\leftrightarrow$ escala de 40 km

- D = 1 (Nordeste) até D = 1.4 (Pará/Maranhão)

- D ("médio") $\approx$ 1.2